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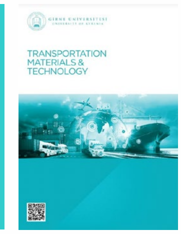

Modeling of internal waves in the Strait of Dardanelles

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ABSTRACT

The tectonic activities, sediment deposits and current erosion govern the particular meandering morphology of the Dardanelles Strait since thousands of years. Later on, invasion of deep Mediterranean waters gave rise to an elongated channel type two-layered water system. Constriction of the sharply bent Nara Passage at the central part could be an additional factor controlling hydraulic flow and mixing. Numerical solution of the corresponding two-dimensional fluid model predicts formation and propagation of internal waves in the form of solitons. They exhibit variability similar to the observed pcynocline (salinity) variations in the water column.

KEYWORDS

Internal waves
Stratified water
Thermocline
Pycnocline
Dardanelles Strait

1. Introduction

From oceanographic point of view, internal waves induced by several physical processes such as winds, tides, gravity, strong currents (Benjamin, 1967; Chen, 1995), propagate at the interior of the water bodies at all depths. Except travelling at the interface of stratified layers, this behavior is somewhat similar to surface waves. Their generation is due to a strong flow, either over a steep bottom topography, or by the protrusion of a sill (Gan and Ingram, 1992) into the well-established layer characterized by a steep temperature/density gradient in the water column (thermocline/pycnocline). Usually, solutions of nonlinear hydrodynamic fluid equations are required for their mathematical prediction. Under certain combinations of the water density gradient represented by the pycnocline (i.e., salinity) and thickness of the interface, internal or solitary waves occur because of buoyancy acting against gravity (Lamb, 1998). The most important features of these waves are the finite amplitude, constant speed and shape, which do not change after collision with another solitary wave. Russell (1844) was the first to identify the formation of such an unchanging single mound as "solitons", generated in the Scottish channel. Most of the proceeding literature was initially concerned with the aspects of a class of nonlinear, non-sinusoidal and somehow isolated waves. Further investigations (e.g., Farmer, 1978; Maxworthy, 1979; Hereman, 2008) with generalizations to two-dimensions exist in the literature (Pierini, 1989; Tomasson, 1991; Guyenne, 2006). Many observations concerning different forms of internal waves exist in certain straits of the world ocean. First identification was the eastward propagation of internal waves in the Strait of Gibraltar (Lacombe and Richez, 1982), followed by those of LaViolette and Arnone (1988) for the same type of waves in the same area. These observations were justified by simultaneous visual observations from aircraft and the space shuttle. For example, the tidal flow over the existing Camarinal Sill being westward, an internal lee wave causes a fluid jump towards the westward side of the sill. After the relaxation and eventual reversal of the westward component of the flow, the disturbance ends up as a wave packet of nonlinear internal waves propagating eastward into the Strait. The most interesting observed feature exhibits the property of traveling waves across the pycnocline. Using images of ERS-1 and ERS-2, some authors (e.g. Liu et al., 1998), identified internal waves in the China Sea. For the latter, upwelling of the Kuroshio Current at the western Pacific Shelf break was responsible for their generation.

Thus, motivation for this work constitutes search of internal waves

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(hereafter IW's) for a similar case study of the Turkish Straits System (TSS) thoroughly modelled before for the Bosphorus (Oğuz et al., 1990) and Dardanelles Straits (Oğuz and Sur, 1989; Statchuk, 2001). For the prediction of internal waves, we used the well-established water column representation at certain density and depth combinations by the KdV theory (Liu, 1988) compared with temperature/salinity measurements at these depths.

2. Hydrodynamic Model for a two-layered channel

An inviscid and incompressible fluid of density ρ is modelled hydrodynamically for a channel type configuration (Choi and Camassa, 1996). For consistency in notation x points along the flow and the vertically positive z direction is opposite to that of gravity. The width-averaged velocity components of the flow along the x, z Cartesian coordinates, are u, w and satisfy continuity and the well-known momentum (Euler) equations in two-dimensions (2D):

$$u_{ix} + w_{iz} = 0 \tag{1}$$

$$u_{it} + u_i u_{ix} + \omega_i u_{iz} = -\frac{p_{ix}}{\rho_i} \tag{2}$$

$$\omega_{it} + u_i \omega_{ix} + \omega_i \omega_{iz} = -\frac{p_{iz}}{\rho_i} - g \tag{3}$$

In the equations (1-3), subscripts denote partial differentiation with respect to time t and the coordinates x, y and z. For a two-layered fluid system, i =1, 2 stand for the upper and lower layers, respectively for which a gravitationally stable stratified configuration, $\rho_1 < \rho_2$ has to be satisfied.

The fluid has a free surface above and a rigid bottom below. Then, for a small displacement ξ of the interface, continuity of normal velocity and pressure at the interface imply:

$$\begin{aligned} &\xi_t + u_1 \xi_x = w_1 \\ &\xi_t + u_2 \xi_x = w_2 \\ &p_1 = p_2 \end{aligned} \tag{4}$$

Different wave solutions obtained from standard linearization of the above equations exist in the literature for a two-layer fluid, in a dimensionless form (Guyenne, 2006). However, even for a weakly nonlinear situation this approach will end up in a non-hydrostatic situation where the Boussinesq approximation, complicated by a vertical shear will not be valid anymore (Lee and Beardsley, 1974).

Application of the correct boundary conditions to the specific linear form of the Equations (1-3) yield internal wave solutions for different depth scales in two-layered fluid systems:

a) Korteweg-de Vries equation (KdV) for a shallow layer:

b) Intermediate Long Wave (ILW) for a deep layer

c) Benjamin-Ono (BO) equation for an infinitely deep layer

3. Nonlinear model for shallow water

In the continuity Eq. (1), if the thicknesses of the two layers are much smaller than the channel length L, then rescaling u and w permit a dimensionless representation of the physical variables.

Imposing the correct boundary conditions and integrating across the fluid layer yield:

$$\xi_t + c_0 \xi_x + c_1 \xi \xi_x + c_2 \xi_{xxx} = 0 \tag{5}$$

Eq. (5) is an extended KdV (EKdV), in cubic form with respect to the interface parameter ξ and dispersion coefficients c_1 and c_2:

$$c_0 = \sqrt{\frac{g h_1 h_2 (\rho_2 - \rho_1)}{\rho_1 h_2 + \rho_2 h_1}} \tag{6}$$

$$c_1 = -\frac{3c_0(\rho_1 h_2^2 - \rho_2 h_1^2)}{2(\rho_1 h_2^2 + \rho_2 h_1^2 h_2)} \tag{7}$$

$$c_2 = \frac{c_0(\rho_1 h_1^2 h_2 + \rho_2 h_1 h_2^2)}{6(\rho_1 h_2 + \rho_2 h_1)} \tag{8}$$

c_1 vanishes at a critical depth ratio:

$$\left(\frac{h_1}{h_2}\right)_c = \sqrt{\frac{\rho_1}{\rho_2}} \tag{9}$$

Nonlinear effects are accounted by the modified KdV equation. Solitary wave solutions with different polarities exist depending on whether h_1/h_2 is smaller, greater or equal to the critical depth. Indeed, this occurs at certain density ratios varying as a function of salinity.

4. Nonlinear model for a deep layer

For the configuration of a surface layer laying on top of a deep layer, nonlinear dispersive effects depend on depth ratios and wavelength, according to whether it is a weakly or fully nonlinear situation.

The generalized case is the well-known Benjamin-Ono (hereafter, BO) equation (Benjamin, 1967; Osborne and Burch 1980):

$$u_t + D^{1+a}{}_x u_x + u u_x = 0 \tag{10}$$

The extreme values of *a* in the exponent of the differentiation operator D, corresponds to the well-known KdV case for *a* = 1 and to the BO formulation for *a* = 0. Depth mean fluid velocities and interface displacements follow from asymptotic approximations of the fluid equations conserving mass, momentum and energy. For homogeneous fluid layers the energy conservation, well represented by a Hamiltonian and the Green-Nagdi model (Green and Nagdi, 1976), is similar either for a thin surface or for a deep layer (Choi and Camassa, 1999). The completely integrable one-dimensional representation of internal wave velocities for stratified fluids is:

$$u_t + u u_x + H\left[u_{xx}\right] = 0 \tag{11}$$

Similarly, for the interface displacement the appropriate Hamiltonian operator $H\left[\xi_{xx}\right]$ allows a solution. For a unidirectional flow in a long channel of length L and depth h_1 (with $\beta = h_1/L$, $\beta < L$), the weakly nonlinear Boussinesq case for stable long waves yield the following ILW equation (see e.g. Ono, 1975):

$$\xi_t + c_0 \xi_x - \frac{3c_0}{2h_1} \xi \xi_x + \frac{\rho_r c_0 h_1}{2} H\left[\xi_{xx}\right] = 0 \tag{12}$$

$\rho_r = \frac{\rho_2}{\rho_1}$ Defines the degree of stratification as a relative density difference. Then, after rescaling by the amplitude *A* and length *L*, solution of Eq. (12) occurs as a family of solitary wave equations:

$$\xi_s(x) = A / \left[1 + (x/L)^2\right] \tag{13}$$

$$c_0 = \sqrt{\frac{g h_1 h_2 (\rho_2 - \rho_1)}{\rho_1 h_2 + \rho_2 h_1}} \tag{14}$$

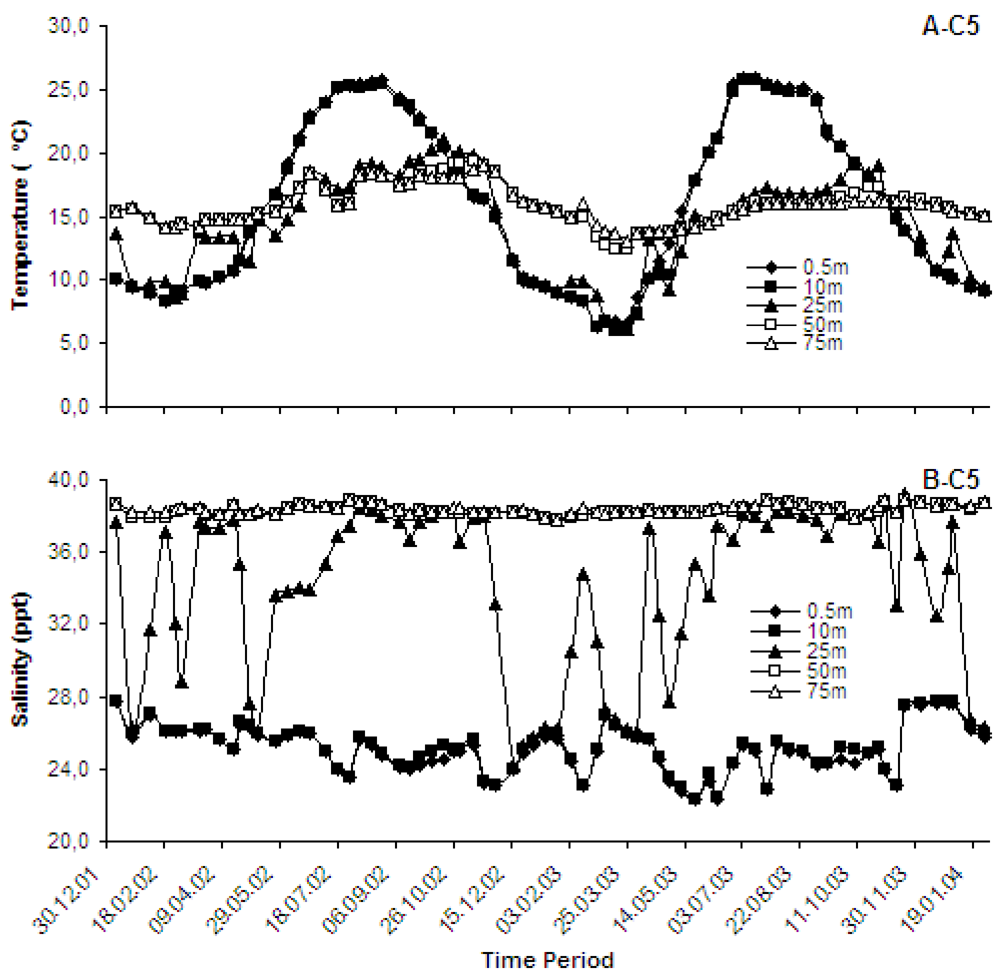

Fig. 1. Temporal variations of temperature (A-C5) and salinity (B-C5) in the northern part of the Nara Passage, Dardanelles (courtesy of Türkoğlu).

with maximum amplitude of:

$$A_m = \frac{h_1 - h_2\sqrt{\frac{\rho_1}{\rho_2}}}{1+\sqrt{\frac{\rho_1}{\rho_2}}} \tag{15}$$

5. Discussions

Under some critical conditions related to the internal hydraulics of a weakly non-linear two-layered stratified fluid, a solitary wave can exist as a solution to the KdV equation (Gan and Ingram, 1992). However, in the case of sharp changes in the bottom topography due to a seamount, shoaling or deepening of the thermocline can affect evolution of these waves. Then, the model predicts generation and evolution of solitary waves along a shoaling thermocline, from nonlinear and dispersive effects separately, at the deep and shallow watersides of the continental shelf. For example Small (2001) also used an extended nonlinear KdV (eKdV) theory for the propagation of long internal waves to show that wave amplitudes shoal or grow depending on depth with a linear speed similar to the Equations (14-15). Zheng et al. (2001) arrive at the same conclusion.

Prediction of internal waves seems plausible from the application of the above modelling procedure to the case of the Nara Passage at the Dardanelles Strait. Unfortunately, the very strong current structure and dense ship crossing in this area make it difficult for instantaneous observations. Türkoğlu and Özcan, (2006) compared their CTD (conductivity, temperature and density) measurements at different times and depths of the Nara Passage with the geographic information system. They reveal the existence of a strong density and temperature stratification, with the seasonal thermocline sometimes oscillating about 20 meters up and separated by a well-mixed wall about 6-7 meters from the deep layer at both sides of the Nara Passage with the following depth and density stratifications:

$$\frac{h_2}{h_1} = 6 - 7 \text{ m}, \quad \frac{\rho_1}{\rho_2} = 1024/1034 - 1027/1038 \tag{18}$$

Here ρ is the density in kg/m^3. Depth ratio of the upper to the lower layer in Eq. (9) is very close to one, which means c_1 is almost zero. Inserting the above values in Eq. (14) and Eq. (15), $c_0 \approx 0.5$ -0.6 m/s for the phase speed and maximum displacement of 0.5 m, which are typical for solitary waves (Figs 1-2).

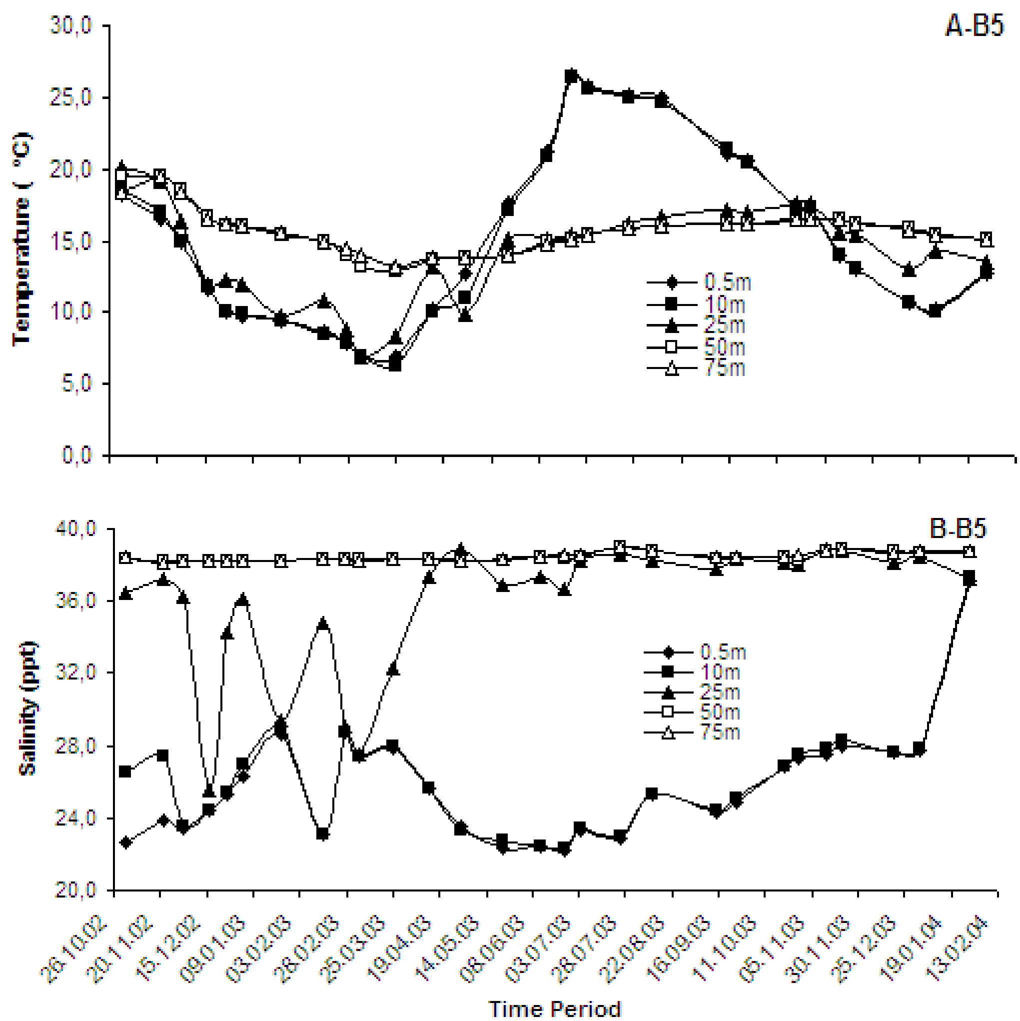

Fig. 2. Temporal variations of temperature (A-B5) and salinity (B-B5) in the southern part of the Nara Passage, Dardanelles (courtesy of Türkoğlu).

6. Conclusions

Kanarska et al. (2001) also conclude on the possibility of a strong mixing at the Nara Passage. From their detailed seismic profiling of the Dardanelles Strait Gökaşan et al. (2008) confirm that Dardanelles has a channel-type structure with 75 km length and 1-8 km width. At the Aegean side a deep sill (about 70 m) or surmount exists reaching about more than 110 m at the Marmara side. They indicate that this sill controls the deep inflow of the Mediterranean waters with a baroclinic forcing created by the salinity difference. From the current meter measurements, they obtain the mean current around 0.514 m/s at the north of the promontory that is comparable to the speed of the solitary waves found above. From the satellite survey of the internal waves in the adjacent Black and Caspian Seas, Lavrova and Mityagina (2017), identify internal waves in the proximity stably stratified ocean surface with a speed of 1 m/s. Therefore, variability of the strong density stratification prevailing in the pycnocline over the particular topography of the Nara Passage may provide hint for a possible triggering mechanism of internal or solitary waves with a possible shoaling at this area. Indeed detailed instantaneous CTD and current profiles combined with the satellite imagery around the Passage could verify this result. On the other hand, identification of IW's in such a non-tidal case by means of satellite data would be rather complicated because of similarity and resemblances in their behavior. Finally, strong mixing due to large amplitude interface waves can also occur at the Bosphorus Strait besides the Dardanelles, as a part of the topographically varying TSS. In his pioneering work Nutku (1984), used the Hamiltonian formulation for a long channel.

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